International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 3, Issue 4, Jul 2014, 25-32

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

ROLL OF DOUBLE SINGLY INFINITE LAPLACE TRANSFORM (SIL-FM RELATION) IN ENGINEERING

N. A. PATIL[1] & VIJAYA N. PATIL[2]

[1]Department of Applied Sciences and Humanities (Mathematics), Shri Sant Gajanan Maharaj College of Engineering, Shegaon, Buldana, Maharashtra, India

[2]Department of Applied Sciences (Mathematics), Padmashri Dr.V.B.Kolte College of Engineering, Malkapur. Buldana, Maharashtra, India

ABSTRACT

The integral transform have played a central role in every aspect of applied mathematics for a very long time and have assumed a greater significance with the advent of the computers and advanced software. There are several transforms which are used to solve differential equations arising in engineering problems. These include Laplace, Mellin, Fourier, and fractional Fourier transform etc.

In this paper, double singly infinite Laplace transform is shown to be equivalent to the singly infinite Laplace-finite Mellin transform (SIL-FM). Then we will focus on the general properties of the singly infinite Laplace-finite Mellin transform and shows how the derivative property is useful for the illustration of Telegraph equation and radio equation using SIL-FM transforms.

KEYWORDS: Double Singly Infinite Laplace Transforms, Mellin Transform, Radio Equation, Singly Infinite Laplace Transforms, Singly Infinite Laplace-Finite Mellin Transform **(**SIL-FM**)**, Telegraph Equation

I. INTRODUCTION

The transforms has received considerable attention from the Engineers. The singly infinite Laplace transform is commonly used to analyse signals, linear models and control systems. In this paper we combine the transforms and apply it on the Transmission line equations.

The outline of this note is as follows –

In section II, we introduce the singly infinite Laplace transforms, Double Singly infinite Laplace transform and Mellin transform. In section III, we show the relation between Transforms (SIL-FM).In section IV, we discussed the general properties of singly infinite Laplace-finite Mellin transforms. In section V, we show the application of derivative property for the illustration of telegraph equation and solution on one of the high-frequency line equation or radio equation using singly infinite Laplace-finite Mellin transforms. In section VI, we conclude.

II. A) SINGLY INFINITE (ONE-SIDED) LAPLACE TRANSFORM

If $f(t)$ is a function defined for all $t \geq 0$,its Laplace transform is an integral of a function $f(t)$ from $t = 0$ to ∞.It is a function of s, and is denoted by $L\left[f(t)\right]$; thus $\overline{F}(s) = L\left[f(t)\right] = \int_0^\infty e^{-st} f(t)\,dt$ (1)

Provided that the integral exists, ‘*s*’ is a parameter which may be real or complex number. Here the upper limit in the integral is infinite; hence it is an improper integral.

Double Singly Infinite Laplace Transform

The Double Singly Infinite Laplace Transform is denoted by

$$L^2\left[f(x,y),s,0,\infty,r,0,\infty\right]=\int_0^\infty\int_0^\infty e^{-(sx+ry)}f(x,y)\,dx\,dy \tag{2}$$

When ever this double integral is exists for r > 0 and s > 0 are parameters

Mellin Transform

Let *f (t)* be a function defined on the positive real axis *0 < t < ∞*. The Mellin transform ‘*M*’ is the operation mapping the function *f* into function *F* defined on the complex plane by the relation-

$$M[f(t),s]=\int_0^\infty t^{s-1}f(t)dt \tag{3}$$

where, ‘s’ is the Mellin parameter.

III. RELATION BETWEEN DOUBLE SINGLY INFINITE (ONE-SIDED) LAPLACE TRANSFORM AND MELLIN TRANSFORMS

In this section we see relation between double Singly Infinite Laplace and Mellin Transform by doing the change of Variable. Double singly infinite Laplace transform is denoted and defined by

$$L^2\left[f(x,y),s,0,\infty,r,0,\infty\right]=\int_0^\infty\int_0^\infty e^{-(sx+ry)}f(x,y)\,dx\,dy \tag{4}$$

$$=\int_0^\infty\int_0^\infty e^{-sx}e^{-ry}f(x,y)\,dx\,dy$$

$$=\int_0^\infty\int_0^\infty e^{-sx}\left(e^{-y}\right)^r f(x,y)\,dx\,dy$$

Substitute y = $-\log\left(\frac{z}{a}\right)$ then $z=ae^{-y}$, $dy=-\frac{dz}{z}$, if y= *0* then z=*a* and if y=∞ then z=*0*

$$L^2\left[f(x,y),s,0,\infty,r,0,\infty\right]=\int_0^\infty\int_a^0 e^{-sx}\left(\frac{z}{a}\right)^r f\left(x,-\log\left(\frac{z}{a}\right)\right)dx\frac{-dz}{z}$$

$$L^2\left[f(x,y),s,0,\infty,r,0,\infty\right]=\int_0^\infty\int_0^a a^{-r}e^{-sx}z^{r-1}f(x,z)\,dx\,dz \tag{5}$$

$$L^2\left[f(x,y),s,0,\infty,r,0,\infty\right]=L^1M_f\left[f(x,z),s,0,\infty,r,0,\text{a}\right]$$

R.H.S. is the singly infinite Laplace- finite Mellin transform for *f(x, z)* in the range [0, ∞], [0, a] and defined by

$$L^1M_f\left[f\left(x,z\right),s,0,\infty,r,0,a\right]=\int_0^\infty\int_0^a a^{-r}e^{-sx}z^{r-1}f\left(x,z\right)dx\,dz \qquad (6)$$

IV] PROPERTIES FOR SINGLY INFINITE LAPLACE- FINITE MELLIN TRANSFORM

P1] Property 1: Linearity Property

Laplace- finite Mellin transform is a linear operator.If α, β, γ be any arbitrary constants and $f\left(x,z\right)$, $g\left(x,z\right)$, $h\left(x,z\right)$ are the functions, then

$$L^1M_f\left[\alpha f\left(x,z\right)+\beta g\left(x,z\right)-\gamma h\left(x,z\right),s,0,\infty,r,0,a\right]$$

$$=\alpha L^1M_f\left[f\left(x,z\right),s,0,\infty,r,0,a\right]+\beta L^1M_f\left[g\left(x,z\right),s,0,\infty,r,0,a\right]-\gamma L^1M_f\left[h\left(x,z\right),s,0,\infty,r,0,a\right] \qquad (7)$$

P2] Property 2: Change of Scale Property

If α, β be any scalar and $f\left(x,z\right)$ is any function, then

$$L^1M_f\left[f\left(\alpha x,\beta z\right),s,0,\infty,r,0,a\right]=\frac{1}{\alpha\beta^r}L^1M_f\left[f\left(p,q\right),\frac{s}{\alpha},0,\infty,r,0,a\beta\right] \qquad (8)$$

Proof: By using definition of Laplace-finite Mellin transform,

$$L^1M_f\left[f\left(\alpha x,\beta z\right),s,0,\infty,r,0,a\right]=\int_0^\infty\int_0^a a^{-r}e^{-sx}z^{r-1}f\left(\alpha x,\beta z\right)dx\,dz$$

Put α $x= p \Longrightarrow, \beta z = q,$

As $x\rightarrow 0$, $p\rightarrow 0$ and as $x \rightarrow\infty, p\rightarrow\infty$

$z\rightarrow 0$, $q\rightarrow 0$ and as z $\rightarrow a$, $q \rightarrow a\beta$

$$L^1M_f\left[f\left(p,q\right),s,0,\infty,r,0,a\beta\right]=\int_0^\infty\int_0^{a\beta} a^{-r}e^{-\frac{sp}{\alpha}}\left(\frac{q}{\beta}\right)^{r-1}f\left(p,q\right)\frac{dp}{\alpha}\frac{dq}{\beta}$$

$$=\frac{1}{\alpha\beta^r}\int_0^\infty\int_0^{a\beta} a^{-r}e^{\left(-\frac{s}{\alpha}\right)p}q^{r-1}f\left(p,q\right)dp\,dq$$

$$=\frac{1}{\alpha\beta^r}L^1M_f\left[f\left(p,q\right),\frac{s}{\alpha},0,\infty,r,0,a\beta\right]$$

P3] Property 3: Power Property

If β be any scalar and $f\left(x,z\right)$ is any function, then

$$L^1M_f\left[f\left(x,z^{\beta}\right),s,0,\infty,r,0,a\right]=\frac{1}{\beta}L^1M_f\left[f\left(x,\mathrm{y}\right),s,0,\infty,\frac{r}{\beta},0,a^{\beta}\right] \tag{9}$$

$$L^1M_f\left[f\left(x,z^{\beta}\right),s,0,\infty,r,0,a\right]=\int_0^{\infty}\int_0^{a}a^{-r}e^{-sx}z^{r-1}f\left(x,z^{\beta}\right)dx\,dz$$

Put, $z^{\beta}=t\Rightarrow z=t^{1/\beta},dz=\frac{1}{\beta}t^{\left(1/\beta\right)-1}dt$

As z→0, $t\to 0$ and as z $\to a$, $t\to a^{\beta}$

$$L^1M_f\left[f\left(x,z^{\beta}\right),s,0,\infty,r,0,a\right]=\int_0^{\infty}\int_0^{a^{\beta}}a^{-r}e^{-sx}\left(\frac{1}{z^{\beta}}\right)^{r-1}f\left(x,t\right)dx\frac{1}{\beta}t^{\left(1/\beta\right)-1}dt$$

$$=\frac{1}{\beta}\int_0^{\infty}\int_0^{a^{\beta}}a^{-r}e^{-sx}\left(t\right)^{\frac{r}{\beta}-1}f\left(x,t\right)dx\,dt$$

$$=\frac{1}{\beta}L^1M_f\left[f\left(x,\mathrm{t}\right),s,0,\infty,\frac{r}{\beta},0,a^{\beta}\right]$$

P4] Property 4: First Shifting Property

$$L^1M_f\left[e^{\alpha x}f\left(x,z\right),s,0,\infty,r,0,a\right]=\int_0^{\infty}\int_0^{a}a^{-r}e^{-sx}z^{r-1}e^{\alpha x}f\left(x,z\right)dx\,dz$$

$$=\int_0^{\infty}\int_0^{a}a^{-r}e^{-(s-\alpha)x}z^{r-1}f\left(x,z\right)dx\,dz$$

$$L^1M_f\left[e^{\alpha x}f\left(x,z\right),s,0,\infty,r,0,a\right]=L^1M_f\left[f\left(x,\mathrm{z}\right),s-\alpha,0,\infty,\mathrm{r},0,a\right] \tag{10}$$

P5] Property 5: Derivative Property

1] $L^1M_f\left[f_x\left(x,z\right),s,0,\infty,r,0,a\right]=\int_0^{\infty}\int_0^{a}a^{-r}e^{-sx}z^{r-1}f_x\left(x,z\right)dx\,dz$

$$=\int_0^{a}a^{-r}z^{r-1}dz\int_0^{\infty}e^{-sx}f_x\left(x,z\right)dx$$

By using integration by parts, we get

$$=\int_0^{a}a^{-r}z^{r-1}dz\left\{\left[e^{-sx}f\left(x,z\right)\right]_0^{\infty}-(-\mathrm{s})\int_0^{\infty}e^{-sx}f\left(x,z\right)dx\right\}$$

$$= (\mathrm{s})\int_0^{\infty}\int_0^{a} a^{-r} e^{-sx} f(x,z) z^{r-1} dx\,dz - k \text{ Where, } k = \int_0^{a} a^{-r} z^{r-1} f(0,z)\,dz$$

$$= (s)L^1 M_f \left[f(x,\mathrm{z}), s, 0, \infty, r, 0, \mathrm{a} \right] - k \tag{11}$$

2) $L^1 M_f \left[f_{xx}(x,z), s, 0, \infty, r, 0, a \right] = \int_0^{\infty}\int_0^{a} a^{-r} e^{-sx} z^{r-1} f_{xx}(x,z)\,dx\,dz$

$$= \int_0^{a} a^{-r} z^{r-1} dz \int_0^{\infty} e^{-sx} f_{xx}(x,z)\,dx$$

By using integration by parts, we get

$$= \int_0^{a} a^{-r} z^{r-1} dz \left\{ \left[e^{-sx} f_x(x,z) \right]_0^{\infty} - (-\mathrm{s}) \int_0^{\infty} e^{-sx} f_x(x,z)\,dx \right\}$$

$$= (\mathrm{s})\int_0^{\infty}\int_0^{a} a^{-r} e^{-sx} f_x(x,z) z^{r-1} dx\,dz - k$$

Where, $\int_0^{a} a^{-r} z^{r-1} f_x(0,z)\,dz = 0$ (By using DUIS)

$$= (s^2)L^1 M_f \left[f(x,\mathrm{z}), s, 0, \infty, r, 0, \mathrm{a} \right] - sk \tag{12}$$

$$L^1 M_f \left[f_{xx}(x,z), s, 0, \infty, r, 0, a \right] = (s^2)L^1 M_f \left[f(x,\mathrm{z}), s, 0, \infty, r, 0, \mathrm{a} \right] - sk$$

Similarly,

$$L^1 M_f \left[f_{xxx}(x,z), s, 0, \infty, r, 0, a \right] = (s^3)L^1 M_f \left[f(x,\mathrm{z}), s, 0, \infty, r, 0, \mathrm{a} \right] - s^2 k \tag{13}$$

In general,

$$L^1 M_f \left[f_x^{\,n}(x,z), s, 0, \infty, r, 0, a \right] = (s^n)L^1 M_f \left[f(x,\mathrm{z}), s, 0, \infty, r, 0, \mathrm{a} \right] - s^{n-1} k \tag{14}$$

V] APPLICATION

In this section we apply the derivative property for the illustration of Telegraph equation and solution on one of the high-frequency line equation or Radio equation using singly infinite Laplace-finite Mellin transforms (SIL-FM).

1] Solving Radio Equation

Equation of transmission line are given by

$$-\frac{\partial v}{\partial x} = \mathrm{R}i + \mathrm{L}\frac{\partial i}{\partial t},\ -\frac{\partial i}{\partial x} = \mathrm{C}\frac{\partial v}{\partial t} + \mathrm{G}v$$

v=voltage in a transmission line

i=current in a transmission line

R =*Resistance and G=conductance to ground*

C= *capacitance to ground*

Since R and G are negligible, ∴ R=G=0, the above equations becomes,

$$-\frac{\partial v}{\partial x} = \mathrm{R}i + \mathrm{L}\frac{\partial i}{\partial t} \qquad (15)$$

$$-\frac{\partial i}{\partial x} = \mathrm{C}\frac{\partial v}{\partial t} \qquad (16)$$

Eliminating *i from* equations (15) and (16),

Differentiating equation (15) partially with respect to *x* and equation (16) partially with respect to *t,* we get

$$-\frac{\partial^2 v}{\partial x^2} = \mathrm{L}\frac{\partial^2 i}{\partial x \partial t} \qquad (17)$$

And $-\frac{\partial^2 i}{\partial x \partial t} = \mathrm{C}\frac{\partial^2 v}{\partial t^2}$ (18)

Using equation (18) in equation(17),we get

$$\frac{\partial^2 v}{\partial x^2} = \mathrm{LC}\frac{\partial^2 v}{\partial t^2} \qquad (19)$$

This is one of the high-frequency line equation or Radio equation similar to one dimensional wave equation. We illustrate this equation by using Laplace-finite Mellin Transform.

The Radio equation is $\frac{\partial^2 v}{\partial x^2} = \mathrm{LC}\frac{\partial^2 v}{\partial t^2}$

Satisfying the initial and boundary conditions

i] If *t=0* then *v(x,0)=0* for all *t*

ii] If *t = l* then *v(x, l)=0* for all *t* (20)

Solution:

$$L^1 M_f\left[v_{xx}(x,z), s, 0, \infty, r, 0, a\right] = \mathrm{LC}\, D_t^{\,2} L^1 M_f\left[v(x,z), s, 0, \infty, r, 0, a\right]$$

$$(s^2)L^1 M_f\left[v(x,\mathrm{z}), s, 0, \infty, r, 0, \mathrm{a}\right] - sk = \mathrm{LC}\, D_t^{\,2} L^1 M_f\left[v(x,z), s, 0, \infty, r, 0, a\right]$$

$$\left(D_t^{\,2} - \frac{s^2}{\mathrm{LC}}\right) L^1 M_f\left[v(x,\mathrm{z}), s, 0, \infty, r, 0, \mathrm{a}\right] = -\frac{sk}{\mathrm{LC}} \qquad (21)$$

This is a linear differential equation with constant coefficient. Its solution is given by

$$L^1M_f\left[v(x,z),s,0,\infty,r,0,a\right]$$

$$=\frac{k}{s}\left\{\left(1-e^{\frac{-2sl}{\sqrt{LC}}}\right)^{-1}\left(1-e^{\frac{-sl}{\sqrt{LC}}}\right)-1\right\}e^{\frac{st}{\sqrt{LC}}}+\frac{k}{s}\left(1-e^{\frac{-2sl}{\sqrt{LC}}}\right)^{-1}\left(e^{\frac{-sl}{\sqrt{LC}}}-1\right)e^{\frac{-st}{\sqrt{LC}}}+\frac{k}{s} \tag{22}$$

Where $k=\int_0^a a^{-r}t^{r-1}v(0,t)\,dt$

2] Solving Telegraph Equation

$$\left.\begin{aligned}\frac{\partial^2 v}{\partial x^2}&=\text{RC}\,\frac{\partial v}{\partial t}\\ \frac{\partial^2 i}{\partial x^2}&=\text{RC}\,\frac{\partial i}{\partial t}\end{aligned}\right\}\text{Telegraph equations}$$

We solve $\frac{\partial^2 i}{\partial x^2}=\text{RC}\,\frac{\partial i}{\partial t}$

$$\frac{\partial i}{\partial t}=\frac{1}{\text{RC}}\frac{\partial^2 i}{\partial x^2} \tag{23}$$

Satisfying the initial and boundary conditions,

i] If *x=0* then i*(t,0)=0* for all *x and*

ii] If *x* = *l* then i*(t, l)=0* for *x* (24)

$$L^1M_f\left[i_t(x,z),s,0,\infty,r,0,a\right]=\frac{1}{\text{RC}}\,D_x^{\,2}L^1M_f\left[i(x,z),s,0,\infty,r,0,a\right]$$

$$\left(D_x^{\,2}-\text{RC}s\right)L^1M_f\left[i_t(x,z),s,0,\infty,r,0,a\right]=-\text{RC}k \tag{25}$$

This is a linear differential equation with constant coefficient. Its solution (using Variation of parameter/direct method) with applying the initial and boundary conditions (24), is given by

$$L^1M_f\left[i_t(x,z),s,0,\infty,r,0,a\right]=$$

$$\frac{k}{s}\left\{\left(1-e^{-2\sqrt{\text{RC}s}\,l}\right)^{-1}\left(1-e^{-2\sqrt{\text{RC}s}\,l}\right)-1\right\}e^{\sqrt{\text{RC}s}\,x}+\frac{k}{s}\left(1-e^{-2\sqrt{\text{RC}s}\,l}\right)^{-1}\left(e^{-\sqrt{\text{RC}s}\,l}-1\right)e^{-2\sqrt{\text{RC}s}\,x}+\frac{k}{s} \tag{26}$$

Where, $k=\int_0^a a^{-r}t^{r-1}i(0,t)\,dt$

VI] CONCLUSIONS

In this article we have presented the close relationship between singly Laplace transform and Mellin Transform. The result seems to be equivalent to singly-doubly Laplace-Mellin Transform & to have a potential to increase the practical utility of Laplace-finite Mellin transform especially in Electrical field. The transforms are considered as a tool to make mathematical calculations easier. The singly infinite Laplace-finite Mellin transform (SIL-FM) is the big source for Engineers to illustrate the enhanced problems.

REFERENCES

1. Dr. N. A. Patil,Vijaya N. Patil "Asymptotic Nature Of Mellin Transform In Electrical Field" International Journal of Engineering Research & Technology (IJERT) Vol. 2 Issue 4, April - 2013 ISSN: 2278-0181
2. KH. M. EL-SHENNAWY, senior member, IEEE, G. PH. FIANI, M. B. TAYEL "TRANSIENT WAVEFORM RESPONSE OF DISTRIBUTED LOADED RC THIN FILM STRUCTURE FOR VOLTAGE STEP FUNCTION USING LAPLACE TRANSFORMATION"0-7803-,2707 IEEE.
3. S.M.Khairnar, R.M.Pise, J.N.Salunke "Relation of Finite Mellin Integral Transform with Laplace and Fourier Transforms" Contemporary Engineering Sciences, Vol. 4, 2011, no. 6, 269 - 288
4. S.M.Khairnar, R.M.Pise, J.N.Salunke "APPLICATION OF THE LAPLACE-MELLIN INTEGRAL TRANSFORM TO DIFFERENTIALEQUATION" International journal of scientific and research publications, volume 2,Issue 5 May-2012